# APPLICATION OF MULTI OBJECTIVE FUZZY LINEAR PROGRAMMING IN SUPPLY PRODUCTION PLANNING PROBLEM

**Abstract**

The purpose of this paper is establish the usefulness of newly formed modified s-curve membership function in a limited supply production planning problem with continuous variables. In this respect fuzzy parameters of linear programming are modeled by non-linear membership function such as s-curve function. This paper begins with introduction and construction of modified s-curve membership function and numerical real life example of supply production planning problem is presented. The computational results show that the superiority of the modified s-curve membership function with fuzzy linear programming technique in optimizing individual objective functions compared to non-fuzzy linear programming approach. Furthermore, for the problem considered, the optimal solution helps to conclude that by incorporating fuzziness in a linear programming model in objective function and constraints, provides a better level of satisfactory solution compared to non-fuzzy linear programming.

Key words : Vagueness, S-curve membership function, Degree of satisfaction, Fuzzy linear programming, Decision making.

**Introduction**

In mathematical programming, problems are expressed as optimizing some objective function given certain constraints. So development of methods of solution were directed towards single objective mathematical programs such as the simplex method for linear programming. In applying mathematical programming decision-makers realized that there are real-life problems though that considers multiple objectives. To be able to come up with a model though that can be solved by the developed solution methods for single-objective mathematical programs these multiple objectives must be combined in some way to become one single objective.

Many problem in operations research, decision science, engineering and management have mainly been studied from optimizing points of view. As the decision making is much influenced by the disturbances of a social and economical circumstances, optimization approach is not always the best. It is because under such influences, many problems are ill-structured. Therefore, a satisfaction approach may be much better than an optimization one. In this regards, it is acceptable that the aspiration level on the treated problem is resolved on the base of past experiences and current knowledge possessed by a decision maker, in the case where the aspiration level of a decision maker should be considered to solve a problem from the perspective of satisfaction strategy. Therefore, it is more natural that the vagueness in the fuzzy system denoted by fuzzy numbers by decision maker.

In real-world decision-making processes in engineering and business, decision making theory has become one of the most important fields. It uses the optimization methodology connected with a single criteria, but also satisfying concepts of multiple criteria. Decision processes with multiple criteria deal with human judgment. This is really hard to modeled. The human judgment element is in the area of preferences defined by the decision maker [1]. First attempts to model decision processes with multiple criteria in business and engineering leads to concepts of multi objective fuzzy linear programming [2]. In this approach the decision maker underpins each objective with a number of goals that should be satisfied [3]. Satisfying requires finding a solution to a multi criterion problem, which is preferred, understood and implemented with confidence. The confidence that the best solution has been found is estimated through the ideal solution. That is the solution which optimizes all criteria simultaneously. Since this is practically unattainable a decision maker considers feasible solutions closest t the ideal solution [4].

Various types of membership functions were used in fuzzy linear programming problem and its application such as a linear membership function [5] [6], a tangent type of a membership function [7], an interval linear membership function [8], an exponential membership function [9], inverse tangent membership function [10], logistic type of membership function [11], concave piecewise linear membership function [12] and piecewise linear membership function [13]. As a tangent type, of a membership function, an exponential membership function, and hyperbolic membership function are non-linear function, a fuzzy mathematical programming defined with a non-linear membership function results in a non-linear programming. Usually a linear membership function is employed in order to avoid non-linearity. Nevertheless, there are some difficulties in selecting the solution of a problem written in a linear membership function. Therefore, in this paper a modified s-curve membership function is employed to overcome such deficits which a linear membership function has. Furthermore, S-curve membership function is more flexible enough to describe the vagueness in the fuzzy parameters for the supply production planning problems.

In this paper, the new methodology of modified s-curve membership function using fuzzy linear programming in supply production planning and their applications to decision making are carried out. Especially, fuzzy linear programming based on a vagueness in the fuzzy parameters such as resource variables given by a decision maker is analyzed.

**Modified S-Curve Membership Function**

The modified S-curve membership function is a particular case of the logistic function with specific values of B, C and α. These values are to be found out. This logistic function as given by equation (1) and depicted in Figure 1 is indicated as S-shaped membership function by Gonguen [14] and Zadeh [15].

We define, here, a modified S-curve membership function as follows:

$$\mu(x) = \begin{cases} 1 & x < x^a \\ 0.999 & x = x^a \\ \dfrac{B}{1 + Ce^{\alpha x}} & x^a < x < x^b \\ 0.001 & x = x^b \\ 0 & x > x^b \end{cases} \tag{1}$$

where $\mu$ is the degree of membership function. Notation α determine the shapes of membership function μ(x), where α > 0. The larger parameter α get, the less their vagueness becomes. It is necessary that parameter α, which determine the figures of membership functions, should be heuristically and experientially decided by experts.

Figure 1 shows the modified S-curve. The membership function is redefined as $0.001 \le \mu(x) \le 0.999$. This range is selected because in supply production the revenue and harmful pollution need not be always 100% of the requirement. At the same time the total revenue and the total harmful pollution will not be 0%. Therefore there is a range between $x^a$ and $x^b$ with $0.001 \le \mu(x) \le 0.999$. This concept of range of $\mu(x)$ is used in this research paper.

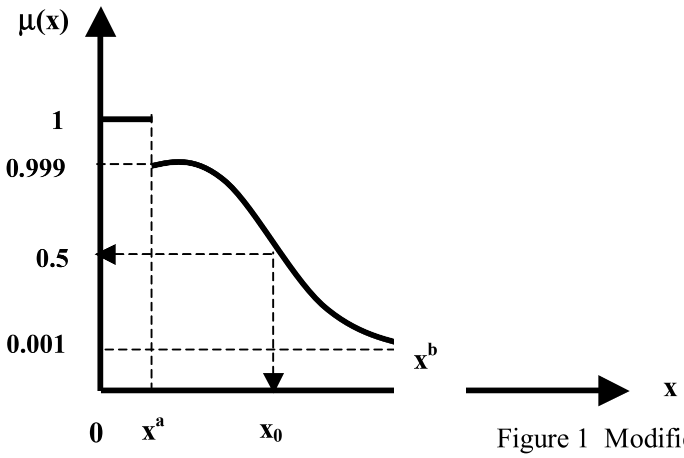


Figure 1 Modified S-Curve Membership Function

We rescale the x axis as $x^a = 0$ and $x^b = 1$ in order to find the values of B, C and α. Novakowska [16] has performed such a rescaling in his work of social sciences.

The values of B, C and α are obtained from equation (1) as

$$B = 0.999\,(1 + C) \tag{2}$$

$$\frac{B}{1 + Ce^{\alpha}} = 0.001 \tag{3}$$

By substituting equation (2) into equation (3) :

$$\frac{0.999(1+C)}{1+Ce^{\alpha}} = 0.001 \tag{4}$$

Rearranging equation (4)

$$\alpha = \ln\frac{1}{0.001}\left(\frac{0.998}{C}+0.999\right) \tag{5}$$

Since, B and α depend on C , we require one more condition to get the values for B, C and α

Let, when $x_0 = \frac{x^a + x^b}{2}$ , $\mu(x_0) = 0.5$ ; Therefore

$$\frac{B}{1+Ce^{\frac{\alpha}{2}}} = 0.5 \tag{6}$$

and hence

$$\alpha = 2\ \ln\left(\frac{2B-1}{C}\right) \tag{7}$$

Substituting equation (5) and equation (6) in to equation (7), we obtain

$$2\ \ln\left(\frac{2(0.999)(1+C)-1}{C}\right) = \ln\frac{1}{0.001}\left(\frac{0.998}{C}+0.999\right) \tag{8}$$

Rearranging equation (8) yields

$$(0.998+1.998C)^2 = C(998+999C) \tag{9}$$

Solving equation (9) :

$$C = \frac{-994.011992 \pm \sqrt{988059.8402 + 3964.127776}}{1990.015992} \tag{10}$$

Since $C$ has to be positive, equation (10) gives $C = 0.001001001$ and from equation (2) and (5), B = 1 and α = 13.81350956.

The modified s-curve membership function has similar shape as the logistic function [11] and the tangent hyperbolic function employed by Leberling [7], but it is more easily handled than the tangent hyperbola. And also a trapezoidal and triangular membership functions are an approximation from a logistic function. Therefore, the s-function is considered much more appropriate to denote a vague goal level which a decision maker considers for the solution implementation. Furthermore, it is possible that the modified s-curve membership function changes its shape according to the parameters values. Then a decision maker is able to apply his strategy to a fuzzy supply production planning using these parameters. Therefore, the modified s-curve membership function is much more convenient than the linear ones.

It should be noted that a triangular or trapezoidal membership functions shows a lower level and a upper level at their membership values 0 and 1 respectively, on the other hand concerning a non-linear membership function such as a modified s-curve function a lower level and upper level may be approximated with membership values 0.001 and 0.999, respectively. The idea of this approach is adopted in this paper and due to Watada [11].

**Fuzzy Resource Parameter**

First, we derive the equation for the fuzzy resource parameter. This equations will be use to generate fuzzy values for the respected parameters.

Fuzzy Resource Parameter $\tilde{b}_i$

From equation (1) for an interval $b_i^a < b_i < b_i^b$ ,

$$\mu_{b_i} = \frac{B}{1 + Ce^{\alpha\left(\frac{b_i - b_i^a}{b_i^b - b_i^a}\right)}}$$

Rearranging exponential term

$$e^{\alpha\left(\frac{b_i - b_i^a}{b_i^b - b_i^a}\right)} = \frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right)$$

Taking $\log_e$ both sides

$$\alpha\left(\frac{b_i - b_i^a}{b_i^b - b_i^a}\right) = \ln\frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right)$$

Hence

$$b_i = b_i^a + \left(\frac{b_i^b - b_i^a}{\alpha}\right)\ln\frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right) \tag{11}$$

Since $b_i$ is the fuzzy resource variable in equation (11), it is denoted by $\tilde{b}_i$. Therefore

$$\tilde{b}_i\Big|_{\mu=\mu_{b_i}} = b_i^a + \left(\frac{b_i^b - b_i^a}{\alpha}\right)\ln\frac{1}{C}\left(\frac{B}{\mu_{b_i}} - 1\right) \tag{12}$$

The membership function for $\mu_{b_i}$ and the fuzzy interval, $b_i^0$ *to* $b_i^1$ for $\tilde{b}_i$ is given in Figure 1.

**Fuzzy Multi-Objective Linear Programming**

The limited Supply Production Planning (SPP) problem with continuous variables are stated as :

A certain company has a factory, which produces 3 products. To produce 1 ton of product A it needs 2 ton of material X, 3 ton of material Y, and 4 ton of material Z. To produce 1 ton of product B it needs 8 ton of material X, and 1 ton of material B. To produce 1 ton of product C it needs it needs 4 ton of material X, 4 ton of material Y, and 2 ton of material Z. At current prices, the company expects to sell product A at a rate of 5 million/ton, product B at a rate of 10 million/ton, and product C at a rate 12 million/ton. But during production process, producing 1 ton of product A generates 1 ton of harmful pollution, producing 1 ton of product B generates 2 ton of harmful pollution, and producing 1 ton of product C produces 1 ton of harmful pollution. The company's objectives are maximize total revenue and to minimize total harmful pollution produced.

This problem can be expressed as the following multi-objective linear program:

Max $z_0 = cx$ and Min $z_1 = dx$

s.t. $Ax \le b$, $x \ge 0$

where : $x^T = (x_1, x_2, x_3)$, $c = (5, 10, 12)$, $d = (1, 2, 2)$

$A = [(2,8,4), (3,1,4), (4,0,2)]$, $b^T = (100, 50, 50)$

Solving each objective separately we get :

(a) for Max cx s.t. $Ax \le b$, $x \ge 0$.

$z_o = 200$, $z_1 = 35.7144$

(b) for Min cx s.t. $Ax \le b$, $x \ge 0$.

$z_0 = 0$, $z_1 = 0$

In (a) we get a maximum revenue of 200 million but this produces 35.71 ton of harmful pollution. In (b) we get the minimum 0 ton of harmful pollution but this would mean 0 million of revenue! As we can see, these two objectives are in conflict with each other. When we maximize revenue we increase pollution. When we minimize pollution we decrease revenue. To come up with a compromise solution respect to vagueness and degree of satisfaction, the company define the following goals :

Goal 1 : Must retain at least 75% of maximum revenue (150 million), but would prefer 100% of maximum revenue (200 million).

Goal 2 : Must not exceed 30 ton of total pollution, but prefer that no pollution is produced.

Goal 3 : The range for total revenue and total harmful pollution should be minimum. This range is called as fuzzy band.

These first two goals can be modeled into fuzzy linear programming and modified s-curve membership function.

The fuzzy linear programming model for the above SPP problem is given as :

$$Max \quad \sum_{j=1}^{3} c_j \, x_j$$

Subject to
$$\sum_{i=1}^{4} a_{ij} \, x_j \leq b_i^a + \left[\frac{b_i^b - b_i^a}{\alpha}\right] \ln \frac{1}{C}\left[\frac{B}{\mu_{b_i}} - 1\right] \tag{13}$$

$$where \quad x_j \geq 0, \quad j = 1,2,3., \quad 0 < \mu_{b_i} < 1 \;, \quad 0 < \alpha < \infty .$$

$C = 0.001001001$, B = 1 and $\alpha = 13.81350956$.

In equation (13) , after trade-off between fuzzy resource parameter $\tilde{b}_i$ and non-fuzzy parameters $a_{ij}$ and $c_j$ the best value for the objective function at the fixed level of μ is reached when [9] :

$$\mu = \mu_{a_{ij}} = \mu_{b_i} \;\; for \quad i = 1,2,3,4. \quad ; j = 1,2,3. \tag{14}$$

Solving equation (13) using linear programming technique, we obtain the following result. Series of iterations are carried out in order to obtain the minimum range for the total revenue and minimum range for the total harmful pollution. The fuzzy band for total revenue defined as $\Delta z^R = z^R_{max} - z^R_{min}$ and for the total harmful pollution $\Delta z^P = z^P_{max} - z^P_{min}$ .

Iteration 1

Table 1

| | | | | |
|---|---|---|---|---|
| $c_j$ | 5 | 10 | 12 | $b_i$ |
| $a_{ij}$ | 2 | 8 | 4 | 100 |
| | 3 | 1 | 4 | 50 |
| | 4 | 0 | 2 | 50 |
| | 1 | 2 | 2 | [0,30) |

Minimum revenue = 33.333 and Maximum revenue = 173. 3333, $\Delta z^R = 140.0$

Iteration 2

Table 2

| $c_j$ | 1 | 2 | 2 | $b_i$ |
|---|---|---|---|---|
| $a_{ij}$ | 2 | 8 | 4 | 100 |
| | 3 | 1 | 4 | 50 |
| | 4 | 0 | 2 | 50 |
| | 5 | 10 | 12 | [33.33,173.33) |

Minimum pollution = 15.2170 and Maximum pollution = 33.1743, $\Delta z^P = 17.9573$

Iteration 3

Table 3

| $c_j$ | 5 | 10 | 12 | $b_i$ |
|---|---|---|---|---|
| $a_{ij}$ | 2 | 8 | 4 | 100 |
| | 3 | 1 | 4 | 50 |
| | 4 | 0 | 2 | 50 |
| | 1 | 2 | 2 | [15.2,33.2) |

Minimum revenue = 104.346 and Maximum revenue = 188.1467, $\Delta z^R = 83.8007$

Iteration 4

Table 4

| $c_j$ | 1 | 2 | 2 | $b_i$ |
|---|---|---|---|---|
| $a_{ij}$ | 2 | 8 | 4 | 100 |
| | 3 | 1 | 4 | 50 |
| | 4 | 0 | 2 | 50 |
| | 5 | 10 | 12 | [104.3,188.1) |

Minimum pollution = 24.4799 and Maximum pollution = 34.5854, $\Delta z^P = 10.1055$

Iteration 5

Table 5

| $c_j$ | 5 | 10 | 12 | $b_i$ |
|---|---|---|---|---|
| $a_{ij}$ | 2 | 8 | 4 | 100 |
| | 3 | 1 | 4 | 50 |
| | 4 | 0 | 2 | 50 |
| | 1 | 2 | 2 | [24.5,34.6) |

Minimum revenue = 147.5729 and Maximum revenue = 194.7319, $\Delta z^R = 47.159$

Iteration 6

Table 6

| $c_j$ | 1 | 2 | 2 | $b_i$ |
|---|---|---|---|---|
| $a_{ij}$ | 2 | 8 | 4 | 100 |
| | 3 | 1 | 4 | 50 |
| | 4 | 0 | 2 | 50 |
| | 5 | 10 | 12 | [147.6,194.7) |

Minimum pollution = 30.1182 and Maximum pollution = 35.2126, $\Delta z^P = 5.0944$

We have to stop at iteration 6. This is because the minimum revenue already exceed 30 ton even though the fuzzy band $\Delta z^P = 5.0944$ is smallest, meaning goal 2 is violated. Therefore iteration 5 gives the good enough outcome for the maximum total revenue and minimum total harmful pollution. The result in Table 5 shows that the total maximum revenue is 194.7319 and the total minimum harmful pollution is 24.4799 at 99.9% degree of satisfaction with the vagueness $\alpha = 13.81350956$.

According to Zimmermann [17] and Carlsson [9], the realistic possible solution exist at 50% degree of satisfaction in a fuzzy environment. The following Table 7 and Table 8 provide the fuzzy solutions for optimal revenue and optimal harmful pollution respect to vagueness $\alpha$ and degree of satisfaction.

The following result is obtained for iteration 5.

Table 7 Optimal Pollution and Degree of Satisfaction
($\alpha$ = 13.81350956)

| Degree of Satisfaction /% | Optimal Pollution/ton |
|---|---|
| 0.10 | 34.5854 |
| 5.09 | 32.2602 |
| 10.08 | 31.6768 |
| 15.07 | 31.3134 |
| 20.06 | 31.0392 |
| 25.05 | 30.8124 |
| 30.04 | 30.6141 |
| 35.03 | 30.4340 |
| 40.02 | 30.2654 |
| 45.01 | 30.1036 |
| 50.00 | 29.9452 |
| 54.99 | 29.7867 |
| 59.98 | 29.6250 |
| 64.97 | 29.4564 |
| 69.96 | 29.2762 |
| 74.95 | 29.0780 |
| 79.94 | 28.8512 |
| 84.93 | 28.5769 |
| 89.92 | 28.2135 |
| 94.91 | 27.6301 |
| 99.90 | 24.4799 |

Table 8 Optimal Revenue and Degree of Satisfaction
($\alpha$ = 13.81350956)

| Degree of Satisfaction /% | Optimal Pollution/ton |
|---|---|
| 0.10 | 147.5729 |
| 5.09 | 161.1643 |
| 10.08 | 163.6814 |
| 15.07 | 165.2493 |
| 20.06 | 166.4324 |
| 25.05 | 167.4109 |
| 30.04 | 168.2663 |
| 35.03 | 169.0435 |
| 40.02 | 169.7710 |
| 45.01 | 170.4687 |
| 50.00 | 171.1524 |
| 54.99 | 171.8361 |
| 59.98 | 172.5338 |
| 64.97 | 173.2613 |
| 69.96 | 174.0386 |
| 74.95 | 174.8940 |
| 79.94 | 175.8724 |
| 84.93 | 177.0556 |
| 89.92 | 178.6235 |
| 94.91 | 181.1405 |
| 99.90 | 194.7319 |

From Table 7, we can see that the total harmful pollution at 50% degree of satisfaction is 29.9452 ton, which satisfies goal two. The total revenue at 50% degree of satisfaction is 171.1524 million, which satisfies goal one. This result explains that the optimum and satisfied solution has been obtained. It is possible to obtain the total harmful pollution less than 29.9452 ton if the vagueness is less than 13.81350956. This can happen most likely in a less fuzzy situation. The above results are comparable to Kirsch [18].

## Conclusion

In general, this research work has achieved its objectives in formulating a new form of membership function and in investigating its applications in a limited supply production planning problem. This membership function is a modified form of the general set of S-curve membership functions. The flexibility of this membership function in applying to real world problem has been proved through an analysis.

Based on the analysis of the results of a real world supply production planning problem of maximizing revenue and minimizing harmful pollution, the following conclusions are drawn :

1. Fuzzy Linear Programming (FLP) is simple and suitable tool for multi-objective problems compared to other methods.
2. The model can be extended to any number of objectives by incorporating only one additional constraint in the constraint set for each additional objective function.
3. The model can be extended to any situation not only supply production planning but any field of engineering with little or no modifications.
4. Analysis of results indicated that total revenue and total harmful pollution in FLP are increased by 9.164% and decreased by 20.727% respectively as compared to maximum membership values in the fuzzy decision Linear Programming (LP) model [18].


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Author : Pandian Vasant

Senior Mathematics Lecturer, Department of Mathematics, American Degree Program, Nilai International College, 71800 Nilai, Malaysia.

Postal Address : Nilai International College, No 1. Persiaran College BBN, 71800 Bandar Baru Nilai, Nilai, Malaysia.

E-mail : pandian_vasant@yahoo.com